SENSOR FUSION ALGORITHMS FOR PEDESTRIAN LOCATION

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Abstract. The problem of estimation of pedestrian position based on multiple sensors is presented. Three algorithms based on multirate Extended Kalman Filter are proposed to solve this problem. One of them comprises modification for colored measurement noise. Another one implements Interacting Multiple Model algorithm. The results obtained using the proposed algorithms with experimental data are presented.

Key Words. Extended Kalman Filter, Interacting Multiple Model, colored noise.

1 INTRODUCTION

Autonomous movement of a blind person requires a source of information about the current position. A DGPS receiver seems ideal for this purpose, because it gives position measurements with theoretical accuracy of a few meters. However, in urban environment the GPS signal is often lost or weakened, which leads to unavailability of position information, or at least significant reduction of accuracy. One way to overcome this problem is to use an independent Dead Reckoning navigation system while GPS measurements are not available. A better solution is to integrate measurements of Dead Reckoning sensors with GPS measurements, which may increase the position estimation accuracy even while GPS measurements are available. Such integration may be accomplished in many different ways. Here we present an Extended Kalman Filter, its modification adapted for color measurement noise, and Interacting Multiple Model algorithm.

2 PROBLEM FORMULATION

We will use a simple Dead Reckoning system, consisting of an electronic compass and a pedometer. The pedestrian is wearing the DR system and a GPS receiver. From the sensors we acquire a sequence of observation vectors $z(t)$, where $t = 0, 1, ...$

$$z(t) = \begin{bmatrix} z_\gamma(t) \\ z_\varphi(t) \end{bmatrix}$$

$$z_\gamma(t) = \begin{bmatrix} z_x(t) \\ z_y(t) \end{bmatrix}, \quad z_\varphi(t) = \begin{bmatrix} z_\delta(t) \\ z_\theta(t) \end{bmatrix}$$

where $z_\gamma(t)$ is the GPS position measurement, and $z_\varphi(t)$ is the measurement of the Dead Reckoning sensors, consisting of position increment during a single time step $z_\delta(t)$ and compass measurement $z_\theta(t)$. The position increment is obtained from the pedometer measurements (see Appendix 1). At times when the GPS measurement is not available, the observation vector is reduced to $z_\varphi(t)$.
Let the pedestrian state $p(t)$ be defined in a similar way

$$p(t) = \begin{bmatrix} p_\gamma(t) \\ p_\varphi(t) \end{bmatrix}$$

$$p_\gamma(t) = \begin{bmatrix} p_x(t) \\ p_y(t) \end{bmatrix}, \quad p_\varphi(t) = \begin{bmatrix} p_\delta(t) \\ p_\theta(t) \end{bmatrix}$$

where $p_\gamma(t)$ denotes the true position of the pedestrian, $p_\delta(t)$ denotes the position increment, and $p_\theta(t)$ denotes the orientation of the pedestrian, which is assumed to be the same as the walking direction.
The assumed model of movement is

$$p(t+1) = f\big(p(t)\big) + w(t) \qquad (1)$$

$$p_x(t+1) = p_x(t) + p_\delta(t)\sin p_\theta(t) + w_x(t)$$

$$p_y(t+1) = p_y(t) + p_\delta(t)\cos p_\theta(t) + w_y(t)$$

$$p_\delta(t+1) = p_\delta(t) + w_\delta(t)$$

$$p_\theta(t+1) = p_\theta(t) + w_\theta(t)$$

where f is the nonlinear transition function and and $w(t) = [w_x(t)\ w_y(t)\ w_\delta(t)\ w_\theta(t)]^T$ is a random perturbation modeled as white Gaussian noise described by constant covariance matrix W.
The corresponding observation equation is

1. When the GPS measurement is available:

$$z(t) = p(t) + v(t) \tag{2}$$

2. When the GPS measurement is available:

$$z_\varphi(t) = p_\varphi(t) + v_\varphi(t) \tag{3}$$

where $v(t) = [v_\gamma(t)^T\ v_\varphi(t)^T]^T$ is the vector of measurement error. $v_\gamma(t) = [v_x(t)\ v_y(t)]^T$ is the GPS measurement error and $v_\varphi(t) = [v_\delta(t)\ v_\theta(t)]^T$ contains measurement errors of respective Dead Reckoning sensors. We will assume $v_\varphi(t)$ to be white Gaussian noise described by constant covariance matrix V_φ.
The nature of error $v_\gamma(t)$ will be discussed later.
Our goal is to estimate the true position $p(t)$ of the pedestrian using a sequence of noisy measurements $\{z(1), \ldots, z(t)\}$.

3 SOLUTIONS

3.1 Extended Kalman Filter Algorithm

The first presented sensor fusion algorithm is based on multirate Extended Kalman Filter [1], [2]. We will assume here that GPS measurement error $v_\gamma(t)$ is white Gaussian noise described by covariance matrix $V_\gamma(t)$, which is dependent on the quality of GPS measurement. Thus we can use a single matrix $V(t)$ to define the measurement error covariance, and all the measurement errors are white Gaussian noise. We will also assume that the measurement errors $v(t)$ are independent of perturbations $w(t)$.
Denote by $F(t)$ the state transition matrix obtained by linearization of the function f around the state trajectory $\widehat{p}(t|t)$ yielded by the EKF algorithm

$$F(t) = \left.\frac{\partial f(p)}{\partial p}\right|_{p=\widehat{p}(t|t)} =$$

$$\begin{bmatrix} 1 & 0 & \sin\widehat{p}_\theta(t|t) & \widehat{p}_\delta(t|t)\cos\widehat{p}_\theta(t|t) \\ 0 & 1 & \cos\widehat{p}_\theta(t|t) & -\widehat{p}_\delta(t|t)\sin\widehat{p}_\theta(t|t) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

In the first step of the filter algorithm a new value of the state vector $\widehat{p}(t|t-1)$ and its associated covariance matrix $P(t|t-1)$ are computed, based on the state vector $\widehat{p}(t-1|t-1)$ and its associated covariance matrix $P(t-1|t-1)$

$$\widehat{p}(t|t-1) = f\big(\widehat{p}(t-1|t-1)\big) \tag{5}$$

$$P(t|t-1) = F(t-1)P(t-1|t-1)F^T(t-1) + W \tag{6}$$

Denote

$$P(t|t-1) = \begin{bmatrix} P_\gamma(t|t-1) & P_{\gamma\varphi}(t|t-1) \\ P_{\gamma\varphi}^T(t|t-1) & P_\varphi(t|t-1) \end{bmatrix} \tag{7}$$

In the second step the new measurement vector $z(t)$ is used to compute the vector of innovations $s(t)$ and its associated covariance matrix $S(t)$, and then the Kalman gain $K(t)$ is computed.

1. When the GPS measurement is available:

$$s(t) = z(t) - \widehat{p}(t|t-1) \tag{8}$$

$$S(t) = P(t|t-1) + V(t) \tag{9}$$

$$K(t) = P(t|t-1)S^{-1}(t) \tag{10}$$

2. When the GPS measurement is not available:

$$s(t) = \begin{bmatrix} 0 \\ z_\varphi(t) - \widehat{p}_\varphi(t|t-1) \end{bmatrix} \tag{11}$$

$$S_\varphi(t) = P_\varphi(t|t-1) + V_\varphi \tag{12}$$

$$S(t) = \begin{bmatrix} 0 & 0 \\ 0 & S_\varphi(t) \end{bmatrix} \tag{13}$$

$$K(t) = \begin{bmatrix} 0 & P_{\gamma\varphi}(t|t-1)S_\varphi^{-1}(t) \\ 0 & P_\varphi(t|t-1)S_\varphi^{-1}(t) \end{bmatrix} \tag{14}$$

In the last step the estimate of the state vector $\widehat{p}(t|t)$ and its associated covariance matrix $P(t|t)$ are updated using the computed Kalman gain $K(t)$.

$$\widehat{p}(t|t) = \widehat{p}(t|t-1) + K(t)s(t) \tag{15}$$

$$P(t|t) = \big(I - K(t)\big)P(t|t-1) \tag{16}$$

Derivation of the Kalman gains is presented in [3].

3.2 Extended Kalman Filter Adapted for Colored Measurement Noise

The experiments have shown that in fact the GPS measurement error is not white noise - the errors from different time steps are clearly correlated. Therefore the second presented sensor fusion algorithm is accommodated for this error correlation. The modification introduced here to Extended Kalman Filter is outlined in [1].
We will now define the GPS measurement error $v_\gamma(t)$ as

$$v_\gamma(t+1) = av_\gamma(t) + b_\gamma(t) \tag{17}$$

where scalar a is a constant correlation factor and $b_\gamma(t)$ is white Gaussian noise described by covariance matrix $B_\gamma(t)$, which is dependent on the quality of GPS measurement. Then we can describe the vector of measurement error $v(t)$ as follows

$$v(t+1) = Av(t) + b(t) \tag{18}$$

$$A = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B(t) = \begin{bmatrix} B_\gamma(t) & 0 \\ 0 & V_\varphi \end{bmatrix}$$

where A is a constant transition matrix and $b(t)$ is white Gaussian noise described by covariance matrix $B(t)$.
Let us approximate the state transition equation (1) by

$$p(t+1) = F(t)p(t) + w(t) \tag{19}$$

where $F(t)$ is as defined in (4).
The solution is based on an estimator of the same size as $p(t)$, and of arbitrarily chosen structure:

$$\widehat{p}(t+1) = F(t)\widehat{p}(t) + K(t-1)\big(z(t+1) - F(t)\widehat{p}(t)\big) \tag{20}$$

We will look for a $K(t)$ value that minimizes the resulting covariance $P(t+1)$. From (2), (3), (19) and (20) we obtain (assuming $\widetilde{p}(t) = p(t) - \widehat{p}(t)$)

$$\widetilde{p}(t+1) = \big(I - K(t+1)\big)\big(F(t)\widetilde{p}(t) + w(t)\big) - K(t+1)v(t+1) \tag{21}$$

Connecting with (17-18) and assuming $E(t) = I - K(t)$ we obtain

$$\begin{bmatrix} \widetilde{p}(t+1) \\ v(t+2) \end{bmatrix} = \begin{bmatrix} E(t+1)F(t) & -K(t+1) \\ 0 & A \end{bmatrix} \times \begin{bmatrix} \widetilde{p}(t) \\ v(t+1) \end{bmatrix} + \begin{bmatrix} 0 & E(t+1) \\ I & 0 \end{bmatrix} \begin{bmatrix} b(t+1) \\ w(t) \end{bmatrix} \tag{22}$$

Because this is a linear system of equations stimulated with white Gaussian noise, the state covariance can be determined by recurrence. Therefore we define

$$E\left\{ \begin{bmatrix} \widetilde{p}(t+1) \\ v(t+2) \end{bmatrix} \begin{bmatrix} \widetilde{p}(t+1) \\ v(t+2) \end{bmatrix}^T \right\} = E \begin{bmatrix} \widetilde{p}(t+1)\widetilde{p}^T(t+1) & \widetilde{p}(t+1)v^T(t+2) \\ v(t+2)\widetilde{p}^T(t+1) & v(t+2)v^T(t+2) \end{bmatrix} = \begin{bmatrix} P(t+1) & \Xi(t+1) \\ \Xi^T(t+1) & \Pi(t+2) \end{bmatrix} \tag{23}$$

and obtain

$$\begin{bmatrix} P(t+1) & \Xi(t+1) \\ \Xi^T(t+1) & \Pi(t+2) \end{bmatrix} = \begin{bmatrix} E(t+1)F(t) & -K(t+1) \\ 0 & A \end{bmatrix} \times \begin{bmatrix} P(t) & \Xi(t) \\ \Xi^T(t) & \Pi(t+1) \end{bmatrix} \times \begin{bmatrix} F^T(t)E^T(t+1) & 0 \\ -K^T(t+1) & A^T \end{bmatrix} + \begin{bmatrix} E(t+1)WE^T(t+1) & 0 \\ 0 & B(t+1)B^T(t+1) \end{bmatrix} \tag{24}$$

The above equation may be rewritten as

$$\Pi(t+2) = A\Pi(t+1)A^T + B(t+1)B^T(t+1) \tag{25}$$

$$\Xi(t+1) = E(t+1)F(t)\Xi(t)A^T - K(t+1)\Pi(t+1)A^T \tag{26}$$

$$\begin{aligned} P(t+1) = {} & E(t+1)F(t)P(t)F^T(t)E^T(t+1) - \\ & K(t+1)\Xi^T(t)F^T(t)E^T(t+1) - \\ & E(t+1)F(t)\Xi^T(t)K^T(t+1) + \\ & K(t+1)\Pi(t+1)K^T(t+1) + \\ & E(t+1)WE^T(t+1) \end{aligned} \tag{27}$$

which after substitution gives the following

$$\begin{aligned} P(t+1) = {} & K(t+1)\Big(F(t)P(t)F^T(t) + \Xi^T(t)F^T(t) + \\ & F(t)\Xi(t) + W + \Pi(t+1)\Big)K^T(t+1) - \\ & K(t+1)\Big(F(t)P(t)F^T(t) + \Xi^T(t)F^T(t) + W\Big) - \\ & \Big(F(t)P(t)F^T(t) + F(t)\Xi(t) + W\Big)K^T(t+1) + \\ & F(t)P(t)F^T(t) + W \end{aligned} \tag{28}$$

The above expression for $P(t+1)$ is minimized by assuming the following value of $K(t+1)$ (see Appendix 2)

$$\begin{aligned} K(t+1) = {} & \Big(F(t)P(t)F^T(t) + F(t)\Xi(t) + W\Big) \times \\ & \Big(F(t)P(t)F^T(t) + \Xi^T(t)F^T(t) + \\ & F(t)\Xi(t) + W + \Pi(t+1)\Big)^{-1} \end{aligned} \tag{29}$$

The complete filtration algorithm is presented below. The first step is almost the same as in the previous subsection

$$\widehat{p}(t|t-1) = f\big(\widehat{p}(t-1|t-1)\big) \tag{30}$$

$$P(t|t-1) = F(t-1)P(t-1|t-1)F^T(t-1) + F(t-1)\Xi(t-1) + W \tag{31}$$

Denote

$$P(t|t-1) = \begin{bmatrix} P_\gamma(t|t-1) & P_{\gamma\varphi}(t|t-1) \\ P^T_{\gamma\varphi}(t|t-1) & P_\varphi(t|t-1) \end{bmatrix} \tag{32}$$

In the second step there are two options, as in the previous subsection.

1. When the GPS measurement is available:

$$s(t) = z(t) - \widehat{p}(t|t-1) \tag{33}$$

$$\Pi(t) = A\Pi(t-1)A^T + B(t-1)B^T(t-1) \tag{34}$$

$$S(t) = P(t|t-1) + \Xi^T(t-1)F^T(t-1) + \Pi(t) \tag{35}$$

$$K(t) = P(t|t-1)S^{-1}(t) \tag{36}$$

$$\Xi(t) = E(t)F(t-1)\Xi(t-1)A^T - K(t)\Pi(t)A^T \tag{37}$$

$$P(t|t) = K(t)S(t)K^T(t) - K(t)P^T(t|t-1) - P(t|t-1)K^T(t) + P(t|t-1) - \Xi(t-1)F(t-1) \tag{38}$$

2. When the GPS measurement is not available:

$$s(t) = \begin{bmatrix} 0 \\ z_\varphi(t) - \widehat{p}_\varphi(t|t-1) \end{bmatrix} \tag{39}$$

$$\Pi(t) = \Pi(0) \tag{40}$$

$$S_\varphi(t) = P_\varphi(t|t-1) + V_\varphi \tag{41}$$

$$K(t) = \begin{bmatrix} 0 & P_{\gamma\varphi}(t|t-1)S_\varphi^{-1}(t) \\ 0 & P_\varphi(t|t-1)S_\varphi^{-1}(t) \end{bmatrix} \tag{42}$$

$$\Xi(t) = \Xi(0) \tag{43}$$

$$P(t|t) = \big(I - K(t)\big)P(t|t-1) \tag{44}$$

In the last step only the estimate of the state vector $\widehat{p}(t|t)$ remains to be updated

$$\widehat{p}(t|t) = \widehat{p}(t|t-1) + K(t)s(t) \tag{45}$$

3.3 Interacting Multiple Model Algorithm with Extended Kalman Filters

Another important feature of the considered problem is that during walking several different ”walking modes” can be distinguished, e.g. walking in constant direction with constant speed and making a turn, to name the two that are most obvious. Those modes differ in terms of parameters of the movement model. This fact is addressed by the third presented sensor fusion algorithm, namely the Interacting Multiple Model algorithm [4], [5], which uses one Extended Kalman Filter for each of the considered modes, and mixes their state estimates and covariance matrices according to current probabilities of individual modes.
We use M Extended Kalman Filters of structure identical to presented in 3.1. All the values defined for the Extended Kalman Filter in 3.1 are used here for each of M modes, with mode number indicated by the lower index. The filters may differ in the values of measurement error and perturbation covariance matrices W and $V(t)$ as well as the in form of state transition function f.

Let $\mu_i(t|t)$ denote the probability that mode i is valid in time step t. Let π_{ij} denote the probability of switching from mode i to mode j.
At the beginning of each time step the estimates from the previous step are mixed to determine the initialization estimates for the current time step. First, the predicted mode probability is computed for each of M modes $(1 \le j \le M)$

$$\mu_j(t|t-1) = \sum_i \pi_{ij} \tag{46}$$

The mixing probability $\mu_{i|j}(t)$ denotes the influence of mode i from time step $t-1$ on initialization estimates of mode j in time step t

$$\mu_{i|j}(t) = \frac{\pi_{ij}\mu_i(t-1|t-1)}{\mu_j(t|t-1)} \tag{47}$$

The initialization estimate $\widehat{p}_{0j}(t-1)$ and its corresponding covariance matrix $P_{0j}(t-1)$ are computed as follows

$$\widehat{p}_{0j}(t-1) = \sum_i \widehat{p}_i(t-1|t-1)\mu_{i|j}(t) \tag{48}$$

$$P_{0j}(t-1) = \sum_i \Big(P_i(t-1|t-1) + \big[\widehat{p}_i(t-1|t-1) - \widehat{p}_{0j}(t-1)\big] \times \big[\widehat{p}_i(t-1|t-1) - \widehat{p}_{0j}(t-1)\big]^T\Big)\mu_{i|j}(t) \tag{49}$$

Each of the pairs $\widehat{p}_{0j}(t-1)$ and $P_{0j}(t-1)$ is then used as an input to the filter of mode j, instead of $\widehat{p}_j(t-1|t-1)$ and $P_j(t-1|t-1)$. The filter output is the estimate vector $\widehat{p}_j(t|t)$ with covariance matrix $P_j(t|t)$ and the innovation vector $s_j(t)$ with covariance matrix $S_j(t)$.
Next the likelihood function $\Lambda_j(t)$ and the new probability $\mu_i(t|t)$ of each mode are computed

$$\Lambda_j(t) = \Big(2\pi \det S_j(t)\Big)^{-0.5} \times \exp\Big(-0.5 s_j^T(t)S_j^{-1}(t)s_j(t)\Big) \tag{50}$$

$$\mu_j(t|t) = \frac{\mu_j(t|t-1)\Lambda_j(t)}{\sum_i \mu_i(t|t-1)\Lambda_i(t)} \tag{51}$$

In the final step the estimates $\widehat{p}_j(t|t)$ and covariance matrices $P_j(t|t)$ from each filter are combined into single estimate $\widehat{p}(t|t)$ and its covariance matrix $P(t|t)$

$$\widehat{p}(t|t) = \sum_j \widehat{p}_j(t|t)\mu_j(t) \tag{52}$$

$$P(t|t) = \sum_j \Big(P_j(t|t) + \big[\widehat{p}_j(t|t) - \widehat{p}(t|t)\big] \times \big[\widehat{p}_j(t|t) - \widehat{p}(t|t)\big]^T\Big)\mu_j(t) \tag{53}$$

4 EXPERIMENTAL RESULTS

Data gathered during three measuring sessions in real environment was used to assess the performance of each sensor fusion algorithm. The data was acquired with time step of one second. Each of the sessions covered somewhat different conditions.

Table 1. Measuring sessions.

Sess.	Duration	Distance covered	GPS availab.	DGPS availab.
1	55 min	5.3 km	95.9%	72.8%
2	80 min	7.8 km	85.9%	77.7%
3	44 min	1.6 km	99.5%	99.5%

Where possible, the parameters for the used algorithms were determined on the basis of measurements. The remaining parameters, e.g. the covariance matrix for perturbation in the movement model, as well as the initial conditions, were assumed arbitrarily, while trying to minimize the position errors. The multiple model algorithm was used with two models corresponding to walking with constant speed and direction and turning. The same parameters were used to process each of the sessions.
The position estimates yielded by each of the presented algorithms, as well as the plain measurements of GPS receiver and Dead Reckoning system, were compared with the reference track in order to obtain the mean square position errors presented in the table below.

Table 2. RMS tracking errors [m].

Sess.	GPS	DR	EKF	EKFc	IMM
1	25.7	35.5	18.8	17,8	17.3
2	52.8	103.6	19.0	18.4	18.1
3	6.7	16.1	7.6	6.3	7.2

High DR error of session 2 is due to the length of this session, which causes higher accumulated errors. High GPS error of session 2 is due to relatively long periods of unavailability of GPS signal. Low GPS error of session 3 is due to long stops, during which the GPS receiver seemed to recover from dynamical errors experienced while walking.
It is clear from the above results that filtration of measurements gives significant improvement of position estimates. The performance level of the presented algorithms does not vary much. However, the results suggest that the EKF algorithm modified for color measurement noise and the IMM algorithm perform better than the plain EKF algorithm. The relation of performance of the modified EKF and IMM algorithms seems to depend on the local conditions.

5 CONCLUSIONS

The problem of sensor fusion for estimation of position of a walking person is presented. A simple model of movement of a walking person is introduced. Three filtering algorithms are proposed to solve the presented problem. The first algorithm is based on Extended Kalman Filter, the second one is EKF filter adapted for color measurement noise, and the third one is based on Interacting Multiple Model algorithm. Experimental results are presented to enable comparison of performance of the algorithms, which is not much diversified. Further work is necessary to precisely analyze the factors influencing the performance of individual filters in order to improve the precision of position estimation.
The objective of the future work is to integrate the last two of the presented algorithms, as well as to implement other multiple model algorithms.

APPENDIX 1 (conversion of pedometer measurements to position increment measurements)

The following algorithm yields the position increment measurements $z_\delta(t)$, assuming the time step length of one second. The pedometer registers the time of occurrence of each step, which is denoted as $T(t,i)$, where t is the discrete time index and i is the number of the step, counting from the first step made during the time period $< t-1, t)$. Time $T(t,i)$ is measured from $t-1$, therefore $0 \leq T(t,i) < 1$. $n(t)$ is the total number of steps made during the time period $< t-1, t)$. L is the average step length, assumed to be constant. Initial conditions are $\widetilde{T}(0) = 0$, $\widetilde{\delta}(0) = 0$. The following algorithm is executed for each time step t.

1. If $\Big(n(t) = 0\Big)$ then $\widetilde{T}(t) = 0$, $\widetilde{\delta}(t) = 0$
2. If $\Big(n(t) = 1$ and $T(t,1) = 0$ and $\widetilde{T}(t) = 0\Big)$ then $\widetilde{T}(t) = 1$, $\widetilde{\delta}(t) = L$
3. If $\Big(n(t) = 1$ and $\big(T(t,1) > 0$ or $\widetilde{T}(t) > 0\big)\Big)$ then $\widetilde{T}(t) = 1 - T(t,1)$, $\widetilde{\delta}(t) = \frac{\widetilde{T}(t)}{T(t,1)+\widetilde{T}(t)}L$
4. If $\Big(n(t) > 1\Big)$ then $\widetilde{T}(t) = 1 - T\big(t, n(t)\big)$, $\widetilde{\delta}(t) = \frac{\widetilde{T}(t)}{T\big(t,n(t)\big) - T\big(t,n(t)-1\big)}L$
5. $z_\delta(t) = n(t)L - \widetilde{\delta}(t-1) + \widetilde{\delta}(t)$

APPENDIX 2 (lemma used for minimization)

If K, A, B, and C are all square matrices of the same size, and A is symmetrical, then the expression

$$KAK^T - KB - B^TK^T + C$$

can be minimized in respect of K as follows. The expression may be equated to

$$KAK^T - KB - B^TK^T + C =$$
$$(K-K_0)A(K-K_0)^T + D$$

After multiplication and resolving the resulting set of equations we obtain

$$K_0 = B^T A^{-1}, \quad D = C - K_0 A K_0^T$$

Thus if we assume $K = B^T A^{-1}$ then the initial expression is reduced to $C - B^T A^{-1} B$.

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